INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination
M. Tech. (CS) II year (1st Sem): 2015–2016
Quantum Information Processing and Quantum Computation

Date: 24. 09. 2015   Maximum Marks : 50   Time : 2.5 Hours
Please try to write all the part answers of a question at the same place.

1. (a) If an atom with a magnetic moment $\overrightarrow{m}$ enters a magnetic field $\overrightarrow{B}$, it experiences a force
$$\overrightarrow{F} = \overrightarrow{\nabla}(\overrightarrow{m} \cdot \overrightarrow{B}),$$
where
$$\overrightarrow{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}.$$
From the above formula, explain the results of the Stern-Gerlach experiment.

   (b) What happens when we place a photon counter behind one of the two slits in Young's double-slit experiment?

[6 + 4]

2. (a) Prove that the eigenvalues of a Hermitian operator are all real.
   (b) If A is Hermitian, show that e^{iA} is unitary.

[5 + 5]

3. (a) Starting from the Schrödinger equation
$$i\hbar\frac{d}{dt}\left|\psi(t)\right\rangle = H\left|\psi(t)\right\rangle,$$
show that the time-evolution operator is unitary, under the assumption that the Hamiltonian is time-independent.

   (b) How is the above mechanism used in designing quantum gates?

[7 + 3]

4. (a) What is quantum entanglement?
   (b) Prove formally that $\frac{1}{\sqrt{2}}\left(\left|01\right\rangle + \left|10\right\rangle\right)$ cannot be written as a tensor product of two separable states.
   (c) If A does local measurement on an entangled state shared with B, then the post-measurement state of B is immediately determined (assuming the same basis of projective measurements), even without the actual measurement. Does this violate the special theory of relativity, by allowing faster-than-light travel?

[2 + 4 + 4]

5. (a) Briefly describe the mathematical formulation of quantum teleportation.
   (b) Does quantum teleportation violate the no-cloning theorem?

[6 + 4]

6. (a) Why is super-dense called the inverse of quantum teleportation?
   (b) Name a universal quantum gate and derive the corresponding unitary matrix from the functional definition of the gate.

[4 + 6]