Quantum Computation 28/08/2015

Lecture 4: Quantum Teleportation

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1 Bell States

Bell States are special two bit entangled states. Following are the four Bell states:

1. $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
2. $|\phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$
3. $|\psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$
4. $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

1.1 Example Sum

$$\frac{1}{2}|\phi^+_{AB}\rangle(\alpha|0_C\rangle + \beta|1_C\rangle) + \frac{1}{2}|\phi^-_{AB}\rangle(\alpha|0_C\rangle - \beta|1_C\rangle) + \frac{1}{2}|\psi^+_{AB}\rangle(\beta|0_C\rangle + \alpha|1_C\rangle) + \frac{1}{2}|\psi^+_{AB}\rangle(\beta|0_C\rangle - \alpha|1_C\rangle) = \frac{1}{\sqrt{2}}(\alpha|0_B\rangle + \beta|1_B\rangle)(|00_{AC}\rangle + |11_{AC}\rangle) \tag{1}$$

2 Teleportation

2.1 Definition

It is a technique by which one object in space can disappear from one point and then reappear at another point.

2.2 Description

Alice and Bob met long ago but now live far apart.While together they generated an entangled pair, each taking one qubit of that pair when they separated.Many years later Alice's mission is to send a qubit $|\psi\rangle$ to Bob, while Bob is hiding. Alice does not know the state of the qubit and she can send only classical information to Bob.

2.3 Procedure of Teleportation

Alice has the qubit ψ . let $\psi = \alpha|0\rangle + \beta|1\rangle$. When Alice and Bob met the entangled state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|0_A 0_C + 1_A 1_C)$ was created. Then they became far apart and each one of them had their own qubits.

1. Alice will create a joined state of three particles. The joined state will be
   $\frac{1}{\sqrt{2}}(|0_A 0_C\rangle + |1_A 1_C\rangle) \otimes \frac{1}{\sqrt{2}}(\alpha|0\rangle_B + \beta|1\rangle_B)$
   The joined state of the three particles can be writtenn as the following (refer equation 1)

   $\frac{1}{2}|\phi^+_{AB}\rangle(\alpha|0\rangle_C+\beta|1\rangle_C)+\frac{1}{2}|\phi^-_{AB}\rangle(\alpha|0\rangle_C-\beta|1\rangle_C)+\frac{1}{2}|\psi^+_{AB}\rangle(\beta|0\rangle_C+\alpha|1\rangle_C)+\frac{1}{2}|\psi^-_{AB}\rangle(\beta|0\rangle_C-\alpha|1\rangle_C)$

2. Alice will do a measurement on the two qubits(A,B) of her possesion in the Bell basis. The measurement will collapse into one of the following four states

   (a) $\frac{1}{2}|\phi^+_{AB}\rangle(\alpha|0\rangle_C + \beta|1\rangle_C)$
   (b) $\frac{1}{2}|\phi^-_{AB}\rangle(\alpha|0\rangle_C - \beta|1\rangle_C)$
   (c) $\frac{1}{2}|\psi^+_{AB}\rangle(\beta|0\rangle_C + \alpha|1\rangle_C)$
   (d) $\frac{1}{2}|\psi^-_{AB}\rangle(\beta|0\rangle_C - \alpha|1\rangle_C)$

3. The result of Alice's Bell measurement tells her which of the above four states the system is in. She can now send her result to Bob through a classical channel. Two classical bits can communicate which of the four results she obtained.

4. After Bob receives the message from Alice, he will know which of the four states his particle is in. Using this information, he performs a unitary operation on his particle to transform it to the desired state $\alpha|0\rangle_B + \beta|1\rangle_B$

   (a) If Alice indicates her result is $|\Phi^+\rangle_{AB}$, Bob knows his qubit is already in the desired state and does nothing. This amounts to the trivial unitary operation, the identity operator.
   (b) If the message indicates $|\Phi^-\rangle_{AB}$, Bob would send his qubit through the unitary quantum gate given by the Pauli matrix

   $$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

   (c) If Alice's message corresponds to $|\Psi^+\rangle_{AB}$, Bob applies the gate

   $$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

   (d) Finally, for the remaining case, the appropriate gate is given by

   $$\sigma_3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$