Quantum Computation 21/08/2015

Lecture 3: No Cloning Theorem & Quantum Entanglement

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Theorem 0.1 (No Cloning Theorem) *It is impossible to create a copy of an arbitary unknown quantum state*

Proof. If possible let us assume U is the unitary operator for cloning.

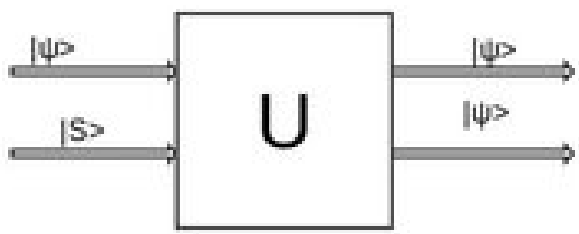

Figure 1: Digaram of cloning a quantum set

From the Diagram it can be observed that

$$U|\psi\rangle|S\rangle = |\psi\rangle|\psi\rangle \tag{1}$$

for $\phi \neq \psi$

$$U|\phi\rangle|S\rangle = |\phi\rangle|\phi\rangle \tag{2}$$

Take the inner product of the both sides of (1) and (2)

$$\begin{aligned}
\langle S|\langle\psi|U^{+}U|\phi\rangle|S\rangle &= \langle\psi|\langle\psi||\phi\rangle|\phi\rangle \\
\langle S|\langle\psi||\phi\rangle|S\rangle &= \langle\psi|\langle\psi||\phi\rangle|\phi\rangle[U^{+}U = I] \\
\langle\psi||\phi\rangle\langle S|S\rangle &= \langle\psi|\phi|\rangle\langle\psi|\phi\rangle[\langle\psi|\phi\rangle isascalarproduct] \\
\langle\psi|\phi\rangle &= \langle\psi|\phi\rangle^2[\langle S|S\rangle = 1]
\end{aligned}$$

let $\langle\psi|\phi\rangle = x$
we have
$x = x^2 \implies$ either $x = 0$ or $x = 1$
$\implies \langle\psi|\phi\rangle = 0$ or 1
$\implies \psi$ and ϕ are either orthogonal or is in the same state
For any two arbitary sates it is not true that they will be orthogonal. Hence it is not possible two perfectly copy any arbitary quantum state □

1 Entanglement

1.1 Definition

The joint state ψ_{AB} of two particles A and B is called separable if ψ_{AB} can be written as a product of the individual states α_A and β_B i.e if $\psi_{AB} = \alpha_A \otimes \beta_B$

The states of two particles are said to be entangled if their joint state is not seperable.

1.2 Example

1. This is an example of a separable state
   $\frac{1}{\sqrt{2}}(|00\rangle_{AB} + |01\rangle_{AB}) = |0_A\rangle \otimes (\frac{1}{\sqrt{2}}(|0_B\rangle + |1_B\rangle))$
2. This is an example of an entangled state
   $\frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 1_B\rangle)$

One may think that it is possible to communicate faster than light using entanglement. But that will violate the special theory of relativity which says no signal can travel faster than light. Suppose there are two particles A and B in the entangled state as mentioned in example 2. Now they are taken far apart (e.g., One in the earth and other in the moon). Now a measurement is done on A. After measurement A can be in state 0 with 50% probability and in state 1 with 50% probability. If A is in state 0 B will automatically collapse in 0 and if A is in state 1 B will automatically collapse in 1 . So B's state can be discovered using the measurement on A only. But there lies a falacy. B can only know his state if A communicates with him to inform about its own state. So classical communication has to happen, which can not be faster than light.